On the Quasiparticle Description of Lattice QCD Thermodynamics[*]

R.A. Schneider[a] and W. Weise[a,b]

[a] *Physik Department, Technische Universität München, D-85747 Garching, GERMANY*

[b] *ECT*, I-38050 Villazzano (Trento), ITALY*

Abstract

We propose a novel quasiparticle interpretation of the equation of state of deconfined QCD at finite temperature. Using appropriate thermal masses, we introduce a phenomenological parametrization of the onset of confinement in the vicinity of the predicted phase transition. Lattice results of the energy density, the pressure and the interaction measure of pure $SU(3)$ gauge theory are excellently reproduced. We find a relationship between the thermal energy density of the Yang-Mills vacuum and the chromomagnetic condensate $\langle \mathbf{B}^2 \rangle_T$. Finally, an extension to QCD with dynamical quarks is discussed. Good agreement with lattice data for 2, 2+1 and 3 flavour QCD is obtained. We also present the QCD equation of state for realistic quark masses.

1 Introduction

There are convincing arguments that QCD exhibits a transition from a confined hadronic phase to a deconfined partonic phase, the quark-gluon plasma (QGP), at a temperature of $T_C \sim 170$ MeV (for two light quark flavours) [1, 2]. A central quantity of matter in thermal equilibrium is the Helmholtz free energy from which the pressure p, energy density ϵ and entropy density s can be derived. These entities are important for the description of, *e.g.*, ultra-relativistic heavy-ion collisions at the CERN SPS and RHIC, or the evolution of the universe after the first $10^{-6} - 10^{-5}$ seconds. The challenge lies therefore in the derivation of the equation of state (EoS) of hot, deconfined QCD from first principles. Perturbative results are available up to order $\mathcal{O}(g_s^5)$ [3]. However, for temperatures of interest in the experimentally accessible region (a few times T_C), the strong coupling constant is presumably large: $g_s \simeq 1.5 - 2$. The perturbative expansion in powers of g_s shows bad convergence already for much smaller values of the coupling. Furthermore, in the vicinity of a phase transition, perturbative methods are in general not expected to be applicable. Non-perturbative methods such as lattice QCD calculations become mandatory. From these numerical simulations the EoS of a pure gluon plasma is known to high accuracy [4, 5], and there are first estimates for the continuum EoS of systems including quarks, albeit still with unphysically large masses [6].

Various interpretations of the lattice data have been attempted in terms of physical quantities, most prominently as the EoS of a gas of non-interacting, massive quark and gluon quasiparticles. Their thermally generated masses are based on perturbative calculations carried out in the hard thermal loop (HTL) scheme [7, 8, 9]. More recently, the QGP has also been described in terms of a condensate of Z_3 Wilson lines [10]. Since all microscopic dynamics has been integrated out in the EoS, there exists no unique interpretation of the lattice data, and one must resort to additional information in order to further restrict the setup of such models. The phenomenological models of the QGP as an ideal gas of massive quarks and gluons have found support from resummed perturbation theory [11] for temperatures $T \gtrsim 3\ T_C$. However, it is astonishing that the lattice EoS is described in this model even close to T_C where one would expect non-perturbative dynamics to enter. Currently there exists not even a qualitative microscopic explanation for this behaviour, and the model has difficulties explaining the dropping of the thermal gluon screening mass in the vicinity of the phase transition.

In this work we extend the quasiparticle approach. Our main new ingredient, as compared to previous work, is a phenomenological parametrization of (de)confinement, supplemented by thermal quasiparticle masses compatible with lattice results. In section II, we shortly review the quasiparticle formalism. Our model is set up, and results for various observables are compared with lattice data of continuum extrapolated SU(3) gauge theory. In section III we give arguments how an extension to a theory including quarks might work, show how the existing lattice data are fitted and estimate the QCD EoS for realistic quark masses. Section IV summarizes and gives an outlook.

2 Gluonic quasiparticles

2.1 Basic quasiparticle model

In this section, we consider an $SU(N_C)$ gluon plasma ($N_C = 3$) at finite temperature and vanishing chemical potential. The use of a quasiparticle model in QCD is based on the observation that in a strongly interacting system, the complex dynamics often rearranges itself in such a way that gross features of the physics can be described in terms of appropriate effective degrees of freedom. From asymptotic freedom, we expect that at very high temperatures the plasma consists of quasifree gluons. As long as the spectral function of the thermal excitations at lower temperatures resembles qualitatively this asymptotic form, a gluonic quasiparticle description is expected to be applicable. The dispersion equation for transverse gluons reads $\omega^2 - k^2 - \Pi_t^*(\omega, k) = 0$. Here, $k = |\vec{k}|$, and Π_t^* is the transverse part of the thermal gluon self-energy. If, for thermal momenta $\omega, k \sim T$, the momentum-dependence of Π_t^* is weak and its imaginary part small, gluon quasiparticles will propagate mainly on-shell with the dispersion relation

$$\omega^2(k) \simeq k^2 + m_g^2(T), \tag{1}$$

where $m_g(T)$ acts as an effective mass generated dynamically by the interaction of the gluons with the heat bath background. Since the existence of a preferred frame of reference breaks Lorentz invariance, new partonic excitations, longitudinal gluonic plasmons, are also present in the plasma. However, their spectral strengths are exponentially suppressed for hard momenta and large temperatures, so gluons are expected to retain their $\nu_g = 2(N_C^2 - 1)$ degrees of freedom despite their masses.

For homogeneous systems of large volume V, the Helmholtz free energy F is related to the pressure p by $F(T, V) = -p(T)V$. In the present framework of a gas of quasiparticles, its explicit expression reads

$$p(T) = \frac{\nu_g}{6\pi^2} \int_0^\infty dk \ f_B(E_k) \frac{k^4}{E_k} - B(T), \tag{2}$$

where ν_g is the gluon degeneracy factor, $E_k = \sqrt{k^2 + m_g^2(T)}$ and

$$f_B(E_k) = \frac{1}{\exp(E_k/T) - 1}. \tag{3}$$

The energy density ϵ and the entropy density s take the form

$$\epsilon(T) = \frac{\nu_g}{2\pi^2} \int_0^\infty dk \ k^2 f_B(E_k) \ E_k + B(T) \tag{4}$$

and

$$s(T) = \frac{\nu_g}{2\pi^2 T} \int_0^\infty dk \ k^2 f_B(E_k) \ \frac{\frac{4}{3}k^2 + m_g^2(T)}{E_k}. \tag{5}$$

The function $B(T)$ is introduced to act as a background field. It is necessary in order to maintain thermodynamic consistency: eqs.(2), (4) and (5) have to satisfy the Gibbs-Duhem relation $\epsilon + p = sT$, and $s(T)$ is related to $p(T)$ via

$$s = \frac{\partial p}{\partial T}. \tag{6}$$

$B(T)$ basically compensates the additional T-derivatives from the temperature-dependent masses in p and is thus not an independent quantity. Since $B(T)$ adds to the energy density of the quasiparticles in eq.(4), it can be interpreted as the thermal vacuum energy density. The entropy density, as a measure of phase space occupation, is unaffected by $B(T)$.

2.2 The HTL model

In previous work [7, 8, 9], the HTL perturbative expression

$$m_g(T) = \sqrt{\frac{N_C}{6}}\, g_s(T)T \quad \text{with} \quad g_s^2(T) = \frac{48\pi^2}{11N_C \ln\left(\frac{T+T_s}{T_C/\lambda}\right)} \tag{7}$$

has been used to model the thermal gluon mass[†]. Here, $m_g(T)$ follows from the transverse part Π_t^* of the thermal gluon polarization tensor in the limit $k \gg g_s T$ and it is gauge-independent to this order. Phenomenology enters in the effective coupling constant $g_s(T)$ with the two fit parameters T_s and λ. By identifying the Landau pole of the effective coupling with a temperature close to T_C, the effective gluon mass (7) becomes very heavy in the vicinity of the phase transition, and s, p and ϵ drop abruptly to almost zero. The resulting EoS of the HTL model is then in good agreement with lattice data over a wide temperature range between T_C and $5\,T_C$ [7]. For temperatures $T \gtrsim 3\,T_C$, HTL resummed perturbation theory [11] indeed supports the picture of the QGP as a gas of weakly interacting, massive quasiparticles. However, there is no *a priori* reason to expect that the one-loop expressions for $g_s(T)$ and Π_t^* can be extrapolated to values of the coupling as large as $g_s(T_C) \sim 2$. Furthermore, close to a phase transition, the reliability of perturbative calculations is questionable. In the light of these facts, it is astonishing how well the HTL model works even in the vicinity of T_C.

Numerical simulations suggest that the deconfinement transition in Yang-Mills theories is second order for $N_C = 2$ [16] and weakly first order for the physical case $N_C = 3$ [17]. From the general theory of critical phenomena, it is expected that the correlation length $\xi(T)$, which is proportional to the inverse of the gluonic screening Debye mass m_D, grows when T_C is approached from above (we recall that m_D measures the exponential decay of the static gluon field correlator $\langle A_0^a(\vec{r})A_0^b(0)\rangle_T \propto \delta^{ab} \exp(-m_D(T)|\vec{r}|)/|\vec{r}|$). For three colours, the mass gap does not vanish at T_C, so $\xi(T_C)$ remains large, but finite. This behaviour is indeed seen in lattice calculations [15] : m_D drops by a factor of ten when going down from $2\,T_C$ to T_C (see figure 1).

In HTL perturbation theory, $m_D(T)$ and $m_g(T)$ are connected by the simple relation:

$$m_D = \sqrt{2}\, m_g. \tag{8}$$

A scenario with heavy masses m_g would then imply *small* correlation lengths close to T_C. It is therefore not clear how a decreasing gluonic Debye mass can be matched to the *heavy, non-interacting* quasiparticles of the HTL model. Of course, in a more general non-perturbative framework, m_g and m_D, although both arise from the same polarization tensor, will not be related *a priori* by a simple constant, as in eq.(8).

[†] In the following, we refer to the quasiparticle model with the gluon mass defined in eq.(7) as the HTL (hard thermal loop) model.

2.3 Deconfinement and the Quasiparticle model

We believe that the failure of simple quasiparticle models to exhibit the correct temperature dependence of the Debye mass can be traced back to the fact that the picture of a non-interacting gas is not appropriate close to T_C because the driving force of the transition, the confinement process, is not taken into account. Below T_C, the relevant degrees of freedom in a pure $SU(3)$ gauge theory are heavy, colour singlet glueballs. Approaching T_C, deconfinement sets in and the gluons are liberated, followed by a sudden increase in entropy and energy density. Conversely, when approaching the phase transition from above, the decrease in the thermodynamic quantities is not caused by masses becoming heavier and heavier, instead the number of thermally active degrees of freedom is reduced due to the onset of confinement. As T comes closer to T_C, an increasing number of gluons gets trapped in glueballs which *disappear* from the thermal spectrum: since $m_{GB} \gtrsim 1.5$ GeV and $T_C \sim 270$ MeV (for pure gauge theory), glueballs are simply too heavy to become thermally excited in the temperature range under consideration (up to about 5 T_C). Of course, glueball masses may also change with temperature. However, since the contribution of glueballs to ϵ, p and s is negligible below T_C (as evident from lattice data), this change is presumably small. The important fact in our opinion is the following: while the confinement mechanism as such is still not understood, it is not necessary to know it in detail since we consider a *statistical* system. All confinement does on a *large* scale is to cut down the number of thermally active gluons as the temperature is lowered. The question is whether this effect of confinement can be reconciled somehow with the quasiparticle picture. We will show in the following that it is indeed possible in a simple, phenomenological way if we allow for an effective, temperature-dependent number of degrees of freedom $\nu_g(T)$.

Let us assume that the thermal gluon mass $m_g(T)$ does not increase as T_C is approached, but instead follows roughly the behaviour of the Debye mass, *i.e.* it decreases. Its detailed T-dependence is not important for the discussion at the moment, but it will be examined in more detail in the next section. Consider now the entropy of a gas of massive gluons along eq.(5) with such a dropping effective gluon mass. The result for $s(T)$ will clearly overshoot the lattice entropy because light masses near T_C lead to an increase in s. However, since the entropy is a measure for the number of active degrees of freedom, the difference may be accounted for by the aforementioned confinement process as it develops when the temperature is lowered toward T_C. This effect can be included in the quasiparticle picture by modifying the number of effective degrees of freedom by a temperature-dependent *confinement factor* $C(T)$:

$$\nu_g \to C(T)\,\nu_g. \tag{9}$$

The replacement (9) leads to the following expressions which replace eqs.(2 - 5):

$$p(T) = \frac{\nu_g}{6\pi^2} \int_0^\infty dk\, [C(T)f_B(E_k)]\, \frac{k^4}{E_k} - B(T), \tag{10}$$

$$\epsilon(T) = \frac{\nu_g}{2\pi} \int_0^\infty dk\, k^2\, [C(T)f_B(E_k)]\, E_k + B(T) \tag{11}$$

and

$$s(T) = \frac{\nu_g}{2\pi^2 T} \int_0^\infty dk\, k^2\, [C(T)f_B(E_k)]\, \frac{\frac{4}{3}k^2 + m_g^2(T)}{E_k}. \tag{12}$$

5

In essence, the factor $C(T)$ represents a statistical parametrization of confinement. Its explicit functional form can be obtained simply as the ratio of the lattice entropy and the entropy (12) calculated with a dropping input gluon mass $m_g(T)$. Qualitatively, we expect $C(T \gg T_C) \approx 1$ at high temperatures where the deviation from the Stefan-Boltzmann limit of ϵ, s and p, as seen on the lattice, is caused solely by the thermal masses m_g. As the phase transition is approached from above, the number of thermally active degrees of freedom decreases and consequently, $C(T)$ becomes less than one. Finally, the entropy below T_C is small, but non-zero, and we can estimate $C(T_C) \sim 0.2$ from lattice data. We would also expect that $C(T)$ is a smooth, monotonously increasing function with T, following the behaviour of the entropy density.

While the lattice entropy is fitted by construction, the crucial test of the model lies now in the reproduction of the energy density ϵ and the pressure p which follow unambiguously from eqs.(11) and (12). Owing to eq.(6), the background field $B(T)$ depends on $m_g(T)$ and $C(T)$ through:

$$
\begin{aligned}
B(T) &= B_1(T) + B_2(T) + B_0, \quad \text{where} \\
B_1(T) &= \frac{\nu_g}{6\pi^2} \int_{T_C}^{T} d\tau \, \frac{dC(\tau)}{d\tau} \int_0^\infty dk \, f_B(E_k) \frac{k^4}{E_k} \quad \text{and} \\
B_2(T) &= -\frac{\nu_g}{4\pi^2} \int_{T_C}^{T} d\tau \, C(\tau) \frac{dm_g^2(\tau)}{d\tau} \int_0^\infty dk \, f_B(E_k) \frac{k^2}{E_k}.
\end{aligned}
\tag{13}
$$

Setting $C(T) = 1$ in B_2 and $B_1 = 0$, one recovers the HTL model expression for $B(T)$. The integration constant B_0 is chosen such that the gluonic pressure equals the very small glueball pressure p_{GB} (which is taken from the lattice) at T_C, according to Gibbs' condition $p_{gluon} = p_{GB}$.

Note that the proposed model for the EoS has no free fit parameters in the sense that once the non-perturbative T-dependence of the thermal gluon mass is fixed, $C(T)$ follows from the ratio of the entropy density of lattice QCD and the entropy density calculated with $m_g(T)$. $B(T)$ is fixed up to an integration constant, which is obtained from Gibbs' condition. Then the energy density and the pressure are uniquely determined. Since the pressure is related to the partition function $\mathcal{Z}$ by $p(T) = -F(T,V)/V = T/V \ln \mathcal{Z}$, the full thermodynamics of the system is known.

2.4 Thermal masses

We must now specify our input thermal quasiparticle mass $m_g(T)$ in eqs.(10 - 13). As mentioned earlier, $m_g(T)$ is to be identified with the transverse part $\Pi_t^*(\omega, k; T)$ of the gluon polarization tensor at $\omega, k \sim T$ (see eq.(1)). Evaluating $m_g(T)$ requires a detailed non-perturbative analysis of the gluonic two-point correlation function which lattice calculations could, in principle, provide. In practice this information does not (yet) exist, so we have to rely on a model.

Suppose we still keep the basic form of eq.(7),

$$
m_g(T) = G(T)T,
\tag{14}
$$

6

but assume that the dimensionless effective coupling $G(T)$ shows approximate critical behaviour at T close to T_C:

$$G(T) \simeq G_0 \left(1 - \frac{T_C}{T}\right)^{\beta}, \tag{15}$$

with some characteristic exponent β and a constant G_0. The assumption (15) implies that the thermal mass behaves as $m_g(T) \sim (T - T_C)^{\beta}$ close to T_C. Asymptotically at $T \gg T_C$, $G(T)$ should match the HTL perturbative form as in (7). In practice we can choose this matching point, for instance, at $T_m = 3\, T_C$. This fixes $G_0 \simeq \sqrt{N_C/6}\, g_s(T_m) \simeq 1.3$. A similar value for T_m below which an explicit HTL resummation is expected to fail, was obtained in ref.[12].

The quantity for which lattice information *does* exist is the Debye screening mass $m_D(T)$ that is related to the longitudinal part $\Pi_l^*(\omega, k; T)$ of the polarization tensor. When defined as $m_D^2 = \Pi_l^*(0, k^2 = -m_D^2)$ [13], the result turns out to be gauge-independent for a wide class of gauges to arbitrary order in perturbation theory [14] (unlike the situation for $\omega = 0$, $k \to 0$ where Π_l^* is not gauge-invariant at next-to-leading order). While there is no *a priori* reason why m_g and m_D should still be related non-perturbatively as they are in perturbation theory (*cf.* eq.(8)), it is nevertheless instructive to recall what is known about the temperature dependence of the Debye screening mass above T_C. Explicit values for m_D have been extracted from lattice calculations of (colour-averaged) heavy quark potentials $V(R, T)$ [15] by the ansatz

$$\frac{V(R, T)}{T} \propto \frac{e^{-\mu(T)R}}{(RT)^d}. \tag{16}$$

Perturbation theory predicts $d = 2$ and $\mu(T) = 2m_D(T)$. As elucidated in [15], the potential may be better reproduced in terms of a mixture of one- and two-gluon exchange since the observed behaviour close to T_C favours values of $d \sim 1.5$ in eq.(16). It is now interesting to observe that the lattice result for $\mu(T)$ can be parametrized very well by

$$\mu(T) \simeq \text{const.} \cdot T \left([1+\delta] - \frac{T_C}{T}\right)^{\beta} \tag{17}$$

with $\beta \simeq 0.1$ and a small gap at $T = T_C$ introduced by $\delta \sim 10^{-6}$. The form of eq.(17) is indeed reminiscent of approximate critical behaviour.

Let us then *assume* that the proportionality (8) between the screening mass m_D and the thermal gluon mass m_g remains at least qualitatively valid in the vicinity of the phase transition, *i.e.* that the exponent β in the characteristic $(1 - T_C/T)^{\beta}$ behaviour of both m_D and m_g is roughly the same. As it turns out, this is *not* a serious assumption: we have checked that, as long as $m_g(T)$ and $m_D(T)$ just have similar trends in their T-evolution close to T_C, our results are not sensitive to the detailed quantitative behaviour of the quasiparticle mass.

Guided by these considerations, the thermal gluon mass m_g is thus parametrized as

$$m_g(T) = G_0 T \left([1+\delta] - \frac{T_C}{T}\right)^{\beta}. \tag{18}$$

where we allow for a small mass gap at $T = T_C$, as indicated by the lattice results for $m_D(T)$ [15]. The small correction $\delta \ll 1$ encodes this deviation. Finally, G_0 is determined by the asymptotic

	G_0	δ	β	C_0	δ_c	β_c
Set A	1.35	10^{-5}	0.2	1.24	0.0029	0.34
Set B	1.30	10^{-6}	0.1	1.25	0.0026	0.31
Set C	1.30	10^{-7}	0.05	1.27	0.0021	0.30

Table 1: Parametrizations for the thermal gluon mass $m_g(T)/T$ and the corresponding confinement factor $C(T)$.

value of the thermal mass, chosen such that the lattice mass and the HTL perturbative result from [8] coincide at $T \approx 3\,T_C$, as mentioned before. In order to account for uncertainties and the approximate nature of relation (18), we have investigated a range of values for G_0, δ and β which can be found in table 1. The upper limit of the range is labelled Set A, an intermediate parameter set Set B and the lower limit Set C. Its plots and the lattice data points for $m_D(T)$ are displayed in figure 1.

A *decreasing* effective coupling strength $G(T)$ as T_C is approached from above, seems at first sight counterintuitive: One would expect that, at a scale $T \sim \Lambda_{QCD}$, 'infrared slavery' sets in, accompanied by an *increasing* QCD coupling g_s. However, it should be borne in mind that this expectation is based on a perturbative result extrapolated to large couplings, neglecting non-perturbative effects. A heuristic argument to make the dropping effective coupling plausible goes as follows. Since we are in a strong coupling regime, the interactions between gluons cannot be described in terms of single gluon exchange, instead they are dominated by (non-perturbative) multi-gluon dynamics. As the temperature is lowered, more and more gluons become confined and form *heavy* glueballs, as outlined earlier. The effective glueball exchange interaction between gluons reduces approximately to a local four-point interaction proportional to $1/m_{GB}^2$. The total interaction can be interpreted as a superposition of multi-gluon and (weak) glueball exchange. Obviously, the more glueballs are formed, the weaker becomes this interaction. The coupling $G(T)$ in eqs.(15) and (18) reflects an interaction between bare gluons from the heat bath on length scales $1/m_{GB}$ and larger that turns these bare gluons into massive, weakly interacting quasiparticles on length scales of order $1/T$.

2.5 Results for SU(3) gauge theory

Now that the temperature behaviour of $m_g(T)$ is given, we can explicitly calculate the entropy density (12). Dividing the lattice entropy density by the result of this calculation, we obtain the T-dependence of the confinement factor $C(T)$. A very good fit of the resulting curves again exhibits an approximate critical power-law behaviour:

$$C(T) = C_0 \left([1 + \delta_c] - \frac{T_C}{T} \right)^{\beta_c}, \qquad (19)$$

which is a non-trivial result. The corresponding parameters of $C(T)$ for the different mass parametrizations, sets A, B and C of (18), can also be found in table 1. Their plots are shown
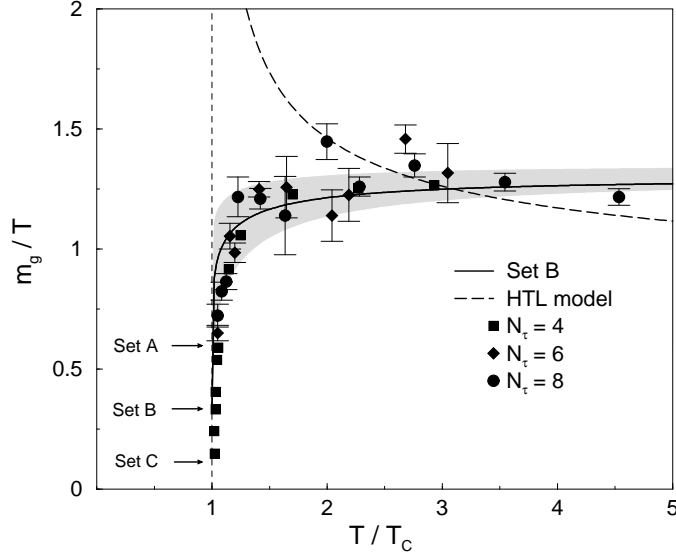
8

Figure 1: The thermal gluon mass m_g/T. The grey band shows the parameter range of table 1 for eq.(18). Set A marks the upper limit and set C the lower, the intermediate set B is displayed as a solid line. The arrows indicate the mass gap $m_g(T_C)/T_C$ of the different sets at the critical temperature. Symbols display $\mu(T)/2$ of eq.(16), with $d = 1.5$ fixed, for lattice configurations of different temporal extent N_τ [15]. For comparison the HTL perturbative mass m_g^{HTL}/T of eq.(7) is also plotted (dashed line), using the parameters of ref.[8].

in figure 2. $C(T)$ is obviously only weakly sensitive to variations of the mass parameters within a broad band, so the further discussion will be based on Set B. In the following, the quasiparticle model with the gluon mass (18) and the confinement factor (19) is referred to as 'confinement model'. For the integration constant B_0 appearing in eq.(13) we find $0.30\ T_C^4 \sim (200\ \mathrm{MeV})^4$. This value is about a factor of 2 larger than in the HTL model and remarkably close to the value of the bag constant at $T = 0$, a welcome feature. As mentioned, we expect the confinement effect to be negligible for $T \gtrsim 3\ T_C$ where the HTL quasiparticle model sets in. From there on, $C(T) \to 1$. The actual deviation of $C(T)$ from 1 for large temperatures has two reasons: first, even in the HTL quasiparticle model a gluon degeneracy of $\nu_g = 16$ does not describe the data, instead a value larger by about 10% is necessary to account for residual sub-leading effects not captured by the model. Second, the behaviour of the gluon mass for larger T is certainly oversimplified since the parametrization $m_g \propto (T - T_C)^\beta$ is expected to be valid only in the vicinity of T_C. Its value overestimates the HTL perturbative result for $T \geq 3\ T_C$ by some 5%, hence the thermodynamical potentials are slightly smaller than in the HTL model. One should instead apply a smooth interpolation between the HTL perturbative mass and the $m_g(T)$ we used for temperatures close to T_C. This would in turn yield a more complicated expression for $C(T)$, but it is in principle straightforward.

In figure 3 we compare results of the confinement model to continuum extrapolated $SU(3)$ lattice data [4]. Obviously, the thermodynamic quantities are very well described even close to
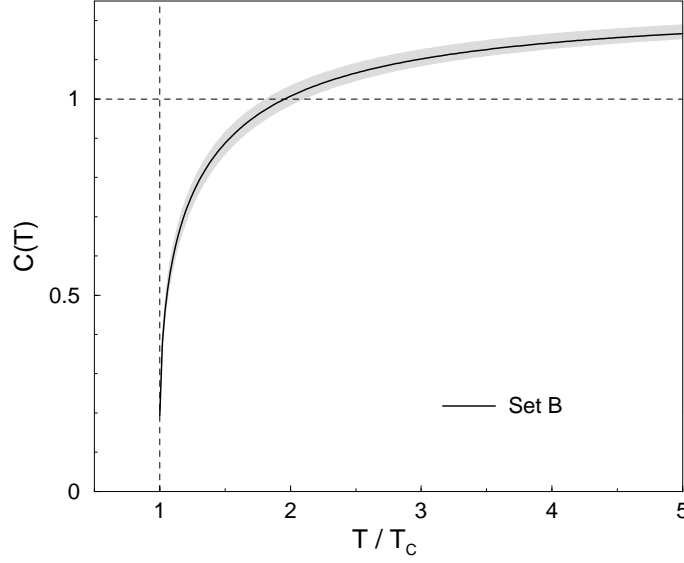
Figure 2: The confinement factor $C(T)$ as a function of temperature. The grey band shows the range for the corresponding mass parametrizations of table 1. The solid line is obtained from set B.

T_C. The slight deviations in the region $\sim 5\,T_C$ arise from our simple parametrization of $C(T)$. We want to stress again that the entropy density is, by construction, *always* fitted. The highly non-trivial test of the proposed model lies in the reproduction of ϵ and p.

A quantity that is sensitive to the finer details of the model is the trace of the energy-momentum tensor, $T^\mu_\mu = \epsilon - 3p$, which is compared to data from a $32^3 \times 8$ lattice in figure 4. The interaction measure

$$\Delta(T) = (\epsilon - 3p)/T^4 \qquad (20)$$

is connected, via the QCD trace anomaly, to the temperature dependent gluon condensate:

$$T^4 \Delta(T) = \langle G^2 \rangle_{T=0} - \langle G^2 \rangle_T. \qquad (21)$$

Here [18, 4],

$$\langle G^2 \rangle_T = \frac{11\alpha_s}{8\pi} \langle G^{a\,2}_{\mu\nu} \rangle_T \quad \text{and} \quad \langle G^2 \rangle_{T=0} = (2.5 \pm 1.0) T_C^4. \qquad (22)$$

Again, excellent agreement over the whole temperature range is observed. The confinement model is even capable of describing the lattice data in the temperature region between T_C and $1.2\,T_C$, where the HTL model significantly underestimates the data.

Finally, figure 5 shows the background field $B(T)$ as a function of temperature. Although the setup of the confinement model is quite different from the HTL model, the shape of this function remains roughly the same. Note, however, that the B_2 term shows a completely different temperature behaviour than in the HTL model: it is monotonously decreasing and negative from T_C on. The B_1 term is vital to reproduce the necessary peak structure. The
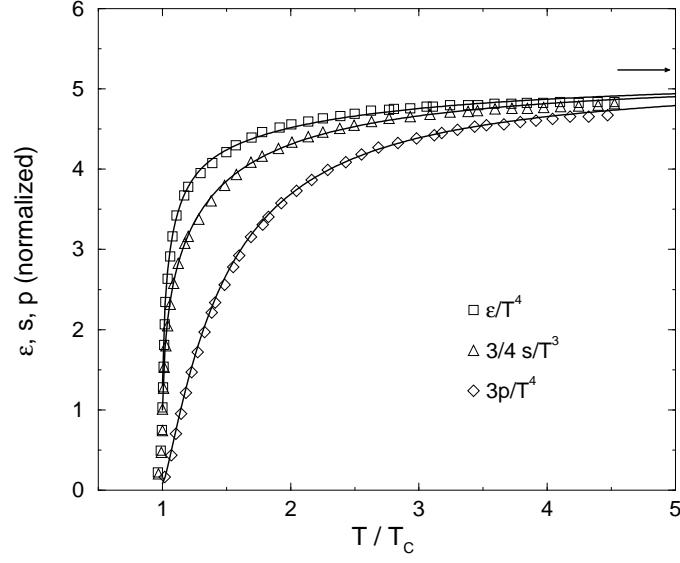
Figure 3: The normalized energy density $\bar{\epsilon} = \epsilon/T^4$, entropy density $\bar{s} = 0.75\ s/T^3$ and pressure $\bar{p} = 3p/T^4$ of our model (solid lines) compared to continuum extrapolated SU(3) lattice data (symbols) [4]. The size of the symbols reflects the lattice uncertainties. The arrow indicates the ideal gas limit for massless gluons.
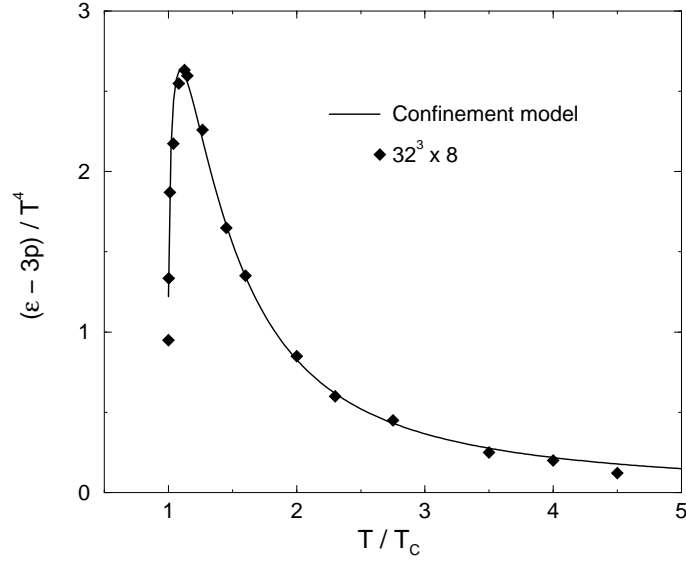

Figure 4: The interaction measure $\Delta = (\epsilon - 3p)/T^4$ of the confinement model (solid line) versus results (symbols) from a $32^3 \times 8$ lattice. The data symbols represent the continuum interpolated values [4].
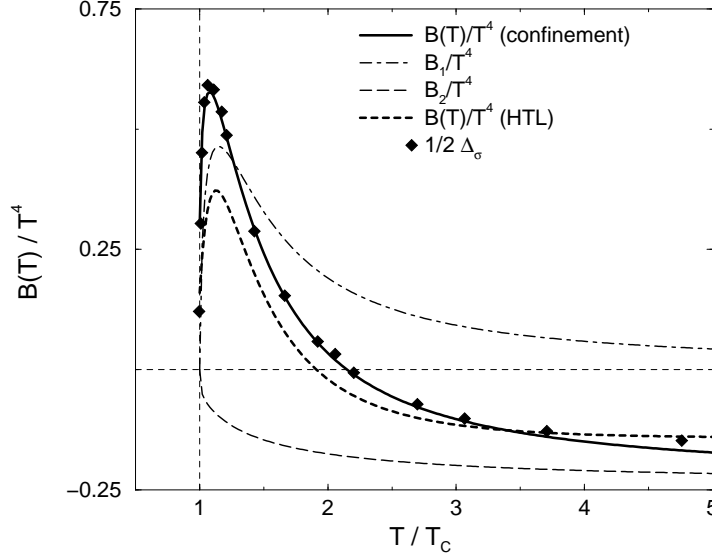
Figure 5: The background field $B(T)$ and its components B_1 and B_2, defined in eq.(13). Also shown is $B(T)$ in the HTL model. Symbols display the spacelike plaquette expectation value $\frac{1}{2}\Delta_\sigma$ taken from the lattice calculation of ref.[4].

maximal value of $B(T)$ is a factor of ~ 1.6 larger than in the HTL model, but it also becomes negative for larger temperatures, with its zero T_0 slightly shifted from $T_0 \sim 2\,T_C$ to $T_0 \sim 2.2\,T_C$. An intriguing observation is that its shape now closely resembles the temperature dependence of the spacelike plaquette expectation value Δ_σ. The space- and timelike plaquettes, Δ_σ and Δ_τ, are related to the interaction measure (20) by $\Delta = \Delta_\sigma + \Delta_\tau$ and can be expressed in terms of the thermal chromomagnetic and chromoelectric condensates $\langle \mathbf{B}^2 \rangle_T$ and $\langle \mathbf{E}^2 \rangle_T$ as

$$
\begin{aligned}
\frac{\alpha_s}{\pi}\langle \mathbf{B}^2 \rangle_T &= -\frac{4}{11}T^4 \Delta_\sigma + \frac{2}{11}\langle G^2 \rangle_{T=0} \quad \text{and} \\
\frac{\alpha_s}{\pi}\langle \mathbf{E}^2 \rangle_T &= \frac{4}{11}T^4 \Delta_\tau - \frac{2}{11}\langle G^2 \rangle_{T=0}.
\end{aligned}
\tag{23}
$$

What we find in fact is $B(T) = \frac{1}{2}\Delta_\sigma(T)$ (see figure 5). This relation between $B(T)$ and $\langle \mathbf{B}^2 \rangle_T$ may be accidental, but it may also hint at a deeper connection between the background field as a carrier of non-perturbative effects, and the magnetic condensate. After all, $B(T)$ represents the thermal energy of the (non-trivial) Yang-Mills vacuum.

3 Quasiparticle model with dynamical quarks

The extension of the mechanism presented in the last section to systems with dynamical quarks is not straightforward. Simulations of fermions on the lattice are still plagued by problems. No concise continuum extrapolation of the QCD EoS with realistic quark masses exists to

12

date. Nevertheless, it is still possible to construct a model of the EoS with quarks, using some reasonable arguments based on the available lattice data.

3.1 Lattice QCD thermodynamics with quarks

There have been lattice calculations of the pressure with different numbers of quark flavours N_f [6, 21]. In the following we focus on results of the Bielefeld group in ref.[6] where a p4-improved staggered action on a $16^3 \times 4$ lattice was used. There, the N_τ-dependence is known to be small, in contrast to the standard staggered fermion actions which show substantially larger cut-off effects. Lattice calculations were performed for two and three flavour QCD with quarks of mass $m_q/T = 0.4$, and for three flavours with two light quarks ($m_q/T = 0.4$) and an additional heavier quark ($m_s/T = 1.0$). From the experience in the pure gauge sector, it has been estimated that the continuum EoS lies about 10 - 20% above the data computed on finite lattices.

Figure 6 displays the lattice pressure, normalized to the Stefan-Boltzmann ideal gas value, for the pure gauge system and for systems with 2, 2+1 and 3 quark flavours. A striking feature is that, within the errors arising from the cut-off dependence, the QCD EoS shows a remarkable flavour *independence* when plotted against T/T_C. This picture suggests that the flavour dependence is well approximated by a term reminiscent of an ideal gas,

$$p(T, N_f) \propto \left(16 + \frac{7}{4} \cdot 2 N_C N_f \right) \frac{\pi^2}{90} \, \tilde{p}(T/T_C) \tag{24}$$

with a function $\tilde{p}(T/T_C)$. Since T_C changes obviously with the number of degrees of freedom present in the thermal system (and therefore with N_f), $\tilde{p}$ is also implicitly N_f-dependent. Scaling against T/T_C, however, the shape of $\tilde{p}$ remains almost the same, indicating that the confinement mechanism itself is only weakly flavour-dependent. Therefore, once we understand the machinery that is responsible for $\tilde{p}(T)$ in the gluon sector, an extrapolation to systems including quarks appears feasible, with the confinement model suitably adapted. Lattice results on the order of the phase transition in full QCD support this idea by indicating that the transition is first order in the case of three light, degenerate quark flavours and most likely second order for two flavours [1, 19, 20]. Note however that, after applying the previously mentioned 10-20% correction to the lattice data, the continuum estimate of the pressure with dynamical quarks is much closer to the ideal gas limit than in the pure gauge sector.

3.2 Thermal masses

No lattice data on thermal masses with dynamical quarks are available. We thus construct effective masses for quarks and gluons by assuming that the N_C- and N_f-dependence of m_g and m_q are both given by the HTL asymptotic limit. For the thermal gluon mass we employ the ansatz:

$$\frac{m_g(T)}{T} = \sqrt{\frac{N_C}{6} + \frac{N_f}{12}} \, \tilde{g}(T, N_C, N_f) \tag{25}$$

with the effective coupling

$$\tilde{g}(T, N_C, N_f) = \frac{g_0}{\sqrt{11 N_C - 2 N_f}} \left([1 + \delta] - \frac{T_C}{T} \right)^\beta . \tag{26}$$
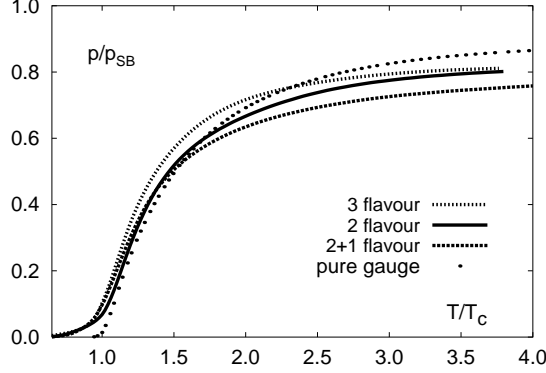
Figure 6: The pressure, normalized to the Stefan-Boltzmann ideal gas value, for the continuum-extrapolated pure gauge system and for systems with 2, 2+1 and 3 flavours on a $16^3 \times 4$ lattice, obtained with a p4-improved staggered fermion action. The continuum limit is estimated to lie about 10 - 20% above the curves shown (figure from [22]).

g_0, δ and β are taken to be universal. Setting $g_0 = 9.4$, $\delta = 10^{-6}$ and $\beta = 0.1$, the gluon and quark masses coincide with the two flavour HTL masses at $T \simeq 3\ T_C$, using the parameters of ref.[7]. The thermal quark mass becomes [7]

$$\frac{m_q(T)}{T} = \sqrt{\left(\frac{m_{q,0}}{T} + \sqrt{\frac{N_C^2 - 1}{16 N_C}} \tilde{g}(T)\right)^2 + \frac{N_C^2 - 1}{16 N_C} \tilde{g}(T)^2} \qquad (27)$$

with the zero-temperature bare quark mass $m_{q,0}$.

In [23], a non-perturbative dispersion equation for a thermal quark interacting with the gluon condensate has been calculated, and it has been found that the effective quark mass is given by $m_q \simeq 1.15\ T$ in the temperature range between $1.1\ T_C$ and $4\ T_C$. Eq.(27) is within 10% in agreement with this result. Nevertheless, very close to T_C the parametrization (27) may be too simple: if the expected chiral phase transition is second order (or weakly first order), fermions may decouple in the vicinity of the phase transition because they have no Matsubara zero modes, and the transition dynamics would be dominated by the bosonic gluons only. In this case gluon masses should become independent of N_f. However, as in the pure gluon sector, the results are stable against small variations of the mass parametrizations, and as long as no further information is available, eqs.(27) and(25) may be taken as an educated guess.

We proceed now as follows: First, we assume that the continuum limit of the pressure can be obtained from the $N_\tau = 4$ lattice data by applying a 10% correction as mentioned, *i.e.* $p_{cont} \simeq 1.1\ p_{lat}$. Second, using Occam's razor we employ a *universal* confinement function $C(T)$ for both quarks and gluons, motivated by eq.(24). The extension of eq.(10) to systems including

	C_0	δ_c	β_c
2 flavours	1.25	0.02	0.28
2+1 flavours	1.16	0.02	0.29
3 flavours	1.03	0.02	0.2
gluon	1.25	0.0026	0.31

Table 2: Parametrizations of eq.(19) for the confinement function $C(T)$ in the presence of dynamical quark flavours. For comparison, the corresponding values of the pure gauge system (set B) are also shown.

quark flavours is straightforward:

$$p(T) = \frac{\nu_g}{6\pi^2} \int_0^\infty dk \left[C(T) f_B(E_k^g)\right] \frac{k^4}{E_k^g} + \sum_{i=1}^{N_f} \frac{2N_C}{3\pi^2} \int_0^\infty dk \left[C(T) f_D(E_k^i)\right] \frac{k^4}{E_k^i} - B(T). \quad (28)$$

Here $E_k^g = \sqrt{k^2 + m_g^2(T)}$ as before, and $f_D(E) = (\exp(E/T) + 1)^{-1}$. The quark energy is $E_k^i = \sqrt{k^2 + m_i^2}$ for each quark flavour $q = i$, and $m_q(T)$ is given by eq.(27) with the bare quark masses $m_{q,0}$. The background field $B(T)$, the entropy density s and the energy density ϵ follow analogously. If the confinement model is applicable, we should expect that the parameters of $C(T)$ in eq.(19), as shown in table 1 for the gluonic calculations, are similar in the presence of quarks. We start therefore with the gluon values for C_0, δ_c and β_c and vary them slightly until good agreement with the lattice pressure is obtained. To account for the temperature-dependent *bare* masses used specifically in the lattice calculations, the quark masses $m_{q,0}$ in eq.(27) are set to $m_{q,0} = 0.4\,T$ (light quarks) and $m_{s,0} = 1.0\,T$ (heavy quark). Figure 7 shows the results for 2 and 2+1 flavours, the corresponding values for the parameters of $C(T)$ can be found in table 2. We observe that indeed, the confinement factor $C(T)$ does not differ much from the pure gluonic case. The factor B_0 is set to $1.4\,T_C^4 \simeq (180\ \mathrm{MeV})^4$. The larger value for δ_c is explained by noting that, for $T < T_C$, many light quark-antiquark composites (pions, kaons etc.) are present. They contribute sizably to the entropy in the hadronic phase. Accordingly, $C(T_C)$ is larger than in the pure gluon case, hence δ_c has to increase.

3.3 Physical quark masses

Pressure, energy and entropy density for physical quark masses are finally obtained by setting $m_{q,0}$ in eq.(27) to the real-world values $m_{u,d} \simeq 0$ and $m_s \simeq 170$ MeV. This procedure assumes that $C(T)$ is independent of $m_{q,0}$ which is not clear. In present lattice simulations, the pions are too heavy, $m_\pi^{lat} \geq 450$ MeV, therefore their contribution to s or p is strongly Boltzmann suppressed. Since

$$\frac{e^{-m_\pi^{lat}/T_C}}{e^{-m_\pi^{phys}/T_C}} \approx \frac{1}{7},$$

future computations with lighter, more realistic pion masses are expected to find a substantially larger pressure and entropy in the hadronic phase below T_C. With this *caveat*, we assume for now that $C(T)$ does not depend on $m_{q,0}$. Figure 8 shows a prediction of ϵ, s and p for massless
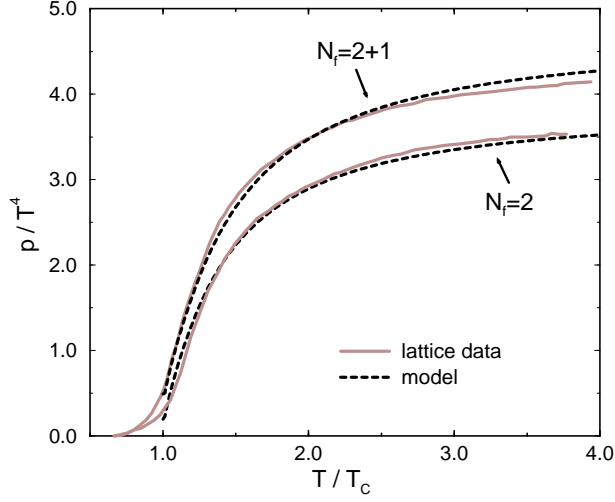
15

Figure 7: The rescaled lattice pressure $p_{cont} \simeq 1.1 p_{lat}$ (grey lines) for 2 and 2+1 flavours and the pressure obtained from the confinement quasiparticle model with running bare quark masses (dashed lines). Values for the parameters of $C(T)$ are shown in table 2.

two flavour QCD. Reassuringly, the pressure of the confinement model is well within the estimate for the continuum EoS of ref.[6] for $T > 2\ T_C$. In contrast to the pure gluon EoS, we observe that the energy and entropy are close to the ideal gas limit already at $T = 3\ T_C$. However, it has to be borne in mind that their normalization is set by C_0 which in turn depends on the continuum estimate of the $N_\tau = 4$ lattice data. More reliable estimates for the continuum pressure are needed to confirm this behaviour. It is also worthwhile noting that, going from temperature-dependent bare masses (as used in the lattice simulations) to the chiral limit, the corresponding change of the pressure in the confinement model is stronger than expected from an ideal Fermi gas. It rises by about 7% whereas for an ideal gas with quark mass $m_q/T = 0.4$ the difference would be only about 3.5% (for $N_f = 2$).

In figure 9, we plot the energy density for three light quark flavours on a $16^3 \times 4$ lattice [22] and as obtained in the confinement model. Since there is no estimate of the continuum limit of these lattice data, the normalization, set by C_0 in eq.(19), is substantially smaller than in previous cases. Apart from that, the data are very well reproduced down to T_C. Finally, figure 10 shows the calculated pressure, energy and entropy density for a system with two light quark flavours ($m_{q,0} = 0$) and a heavier strange quark ($m_{s,0} \simeq 170$ MeV). Here, the approach to the Stefan-Boltzmann limit is obviously slower than in the two flavour case because of the mass suppression of the third, heavier flavour. N_f in eqs.(25), (26) and (27) was set to 2.3. The results for ϵ and p are also in good agreement for $T > 2\ T_C$ with an EoS obtained in the HTL quasiparticle model [24]. Closer to T_C, the confinement model predicts a stronger decrease of the energy density, though. Again, the background field $B(T)$ resembles the shape of the corresponding function in the HTL model. However its zero, T_0, is considerably shifted, from
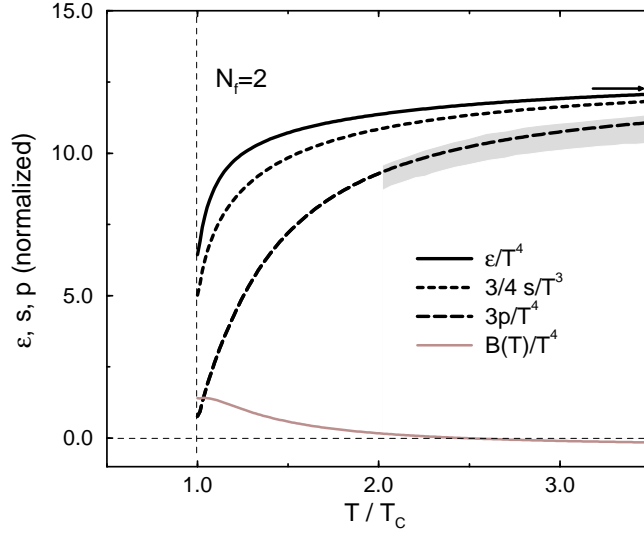
16

Figure 8: Pressure, energy and entropy density for two light quark flavours in the confinement model. The arrow indicates the ideal gas limit. The grey band is an estimate of the continuum EoS for massless two flavour QCD, based on an extrapolation of lattice results [6].

$T_0 \sim 1.7\ T_C$ to $T_0 \sim 2.7\ T_C$.

4 Summary and outlook

We have presented a novel quasiparticle description of the QCD EoS at finite temperature. Our main modification as compared to previous work is the schematic inclusion of confinement thas has so far been neglected. First, we have considered thermal SU(3) gauge theory. Guided by lattice results for the thermal screening masses, we constructed a thermal mass $m_g(T)$ for the gluon and calculated the corresponding entropy density as a measure of phase space occupation. The difference to the lattice entropy was then attributed to the confinement process setting in as T_C is approached from above. Since confinement simply reduces the number of thermally active degrees of freedom in a statistical sense, this non-perturbative behaviour is incorporated in a model of quasifree, massive quasiparticles by a modification of the particle distribution functions with a confinement factor $C(T)$. The energy density and the pressure are then uniquely determined. Our model agrees very well with continuum-extrapolated lattice data. The interaction measure $\Delta(T)$ is even better reproduced than in the HTL model. As a new and interesting aspect, a possible physical connection of the background field $B(T)$ (which is the thermal energy of the Yang-Mills vacuum) with the chromomagnetic condensate $\langle \mathbf{B}^2 \rangle_T$ has been proposed. Our results turn out not to be very sensitive to the detailed parametrization of the T-dependent quasiparticle mass; the driving new feature is the confinement factor $C(T)$.

For systems with dynamical quarks, sufficiently precise lattice data are not yet available. With
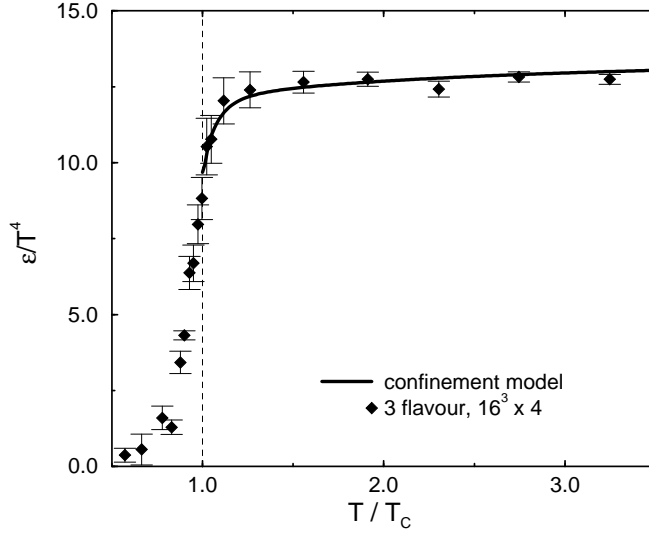
17

Figure 9: Energy density estimate in the chiral limit for three quark flavours on a $16^3 \times 4$ lattice [22] (data points) and as obtained in the confinement model (solid line).
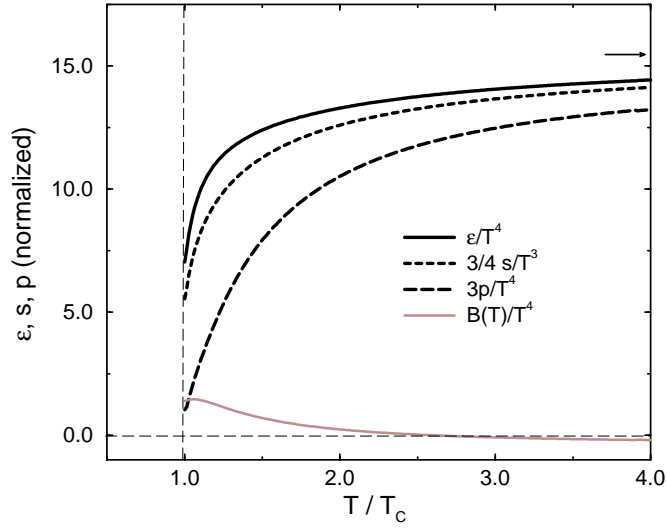

Figure 10: Pressure, energy and entropy density for two light quark flavours ($m_{u,d} = 0$) and a heavier strange quark ($m_s \simeq 170$ MeV) in the confinement model. The arrow indicates the ideal gas limit of massless three-flavour QCD.

reasonable assumptions, we can nevertheless reproduce existing continuum estimates of lattice results for the pressure. Of course, future simulations with higher statistics and smaller quark masses are needed to judge if our confinement model yields sensible results in the presence of quark flavours. It is encouraging, though, that for temperatures $T \gtrsim 2\, T_C$, the predictions of the confinement model with realistic quark masses agree well with HTL model calculations and an estimate of the continuum EoS in the chiral limit.

The function $C(T)$ parametrizes our ignorance about details of the confinement mechanism. It would be desirable to connect this macroscopic quantity with microscopic, first-principle QCD dynamics, preferably starting in the gluon sector. Interesting questions arise in this context, as to the rôle of instantons and condensates of magnetic monopoles. We point out that a comparison of $B(T)$ with lattice data for the spacelike plaquette Δ_σ in the presence of quarks may shed more light on such conjectures.

5   Acknowledgements

References

[1] A. Ali Khan et al. (CP-PACS Collaboration), Phys. Rev. **D63** (2001) 034502.

[2] F. Karsch, A. Peikert and E. Laermann, hep-lat/0012023.

[3] C. Zhai and B. Kastening, Phys. Rev. **D52** (1995) 7232.

[4] G. Boyd et al., Nucl. Phys. **B469** (1996), 419.

[5] M. Okamoto et al. (CP-PACS Collaboration), Phys.Rev. **D60** (1999) 094510.

[6] F. Karsch, A. Peikert and E. Laermann, Phys. Lett. **B478** (2000) 447.

[7] A. Peshier, B. Kämpfer and G. Soff, Phys. Rev. **C61** (2000) 045203.

[8] A. Peshier, B. Kämpfer, O.P. Pavlenko and G. Soff, Phys. Rev. **D54** (1996) 2399.

[9] P. Levai and U. Heinz, Phys. Rev. **C57** (1998) 1879.

[10] R. Pisarski, Phys. Rev. **D62** (2000) 111501.

[11] See the review of J.-P. Blaizot and E. Iancu, hep-ph/0101103, submitted to Phys. Repts.

[12] A. Peshier, Phys. Rev. **D63** (2001) 105004.

[13] A. K. Rebhan, Phys. Rev. **D48** (1993) R3967.

[14] R. Kobe, G. Kunstatter and A. K. Rebhan, Phys. Rev. Lett. **64** (1990) 2992; Nucl. Phys. **B355** (1991) 1.

[15] O. Kaczmarek, F. Karsch, E. Laermann and M. Lütgemeier, Phys. Rev. **D62** (2000) 034021.

[16] F. Karsch, talk given at the 'Enrico Fermi' Int. School of Physics, Varenna, Italy, Jun 26-Jul 6, 1984.

[17] H. Ichie, H. Suganuma and H. Toki, Phys. Rev. **D52** (1995) 2944; B. Beinlich et al., Eur. Phys. J. **C6** (1999) 133.

[18] H. Leutwyler, in Proceedings of the Conference *QCD - 20 years later*, (Edts. P.M. Zerwas and H.A. Kastrup), World Scientific 1993, p. 693.

[19] C. Bernard et al., Phys. Rev. **D61** (2000) 054503.

[20] Y. Iwasaki et al., Phys. Rev. **D54** (1996) 7010.

[21] A. Ali Khan, CP-PACS collaboration, hep-lat/0103028; J. Engels et al., Phys. Lett. **B396** (1997) 210.

[22] F. Karsch, plenary talk given at the 15th International Conference on Ultra-Relativistic Nucleus-Nucleus Collisions (QM 2001), hep-ph/0103314.

[23] A. Schäfer and M. Thoma, Phys. Lett. **B451** (1999) 195.

[24] A. Peshier, talk given at 5th International Workshop on Thermal Field Theories and Their Applications, Regensburg, Germany, 10-14 Aug 1998, hep-ph/9809379.